Gr 10 Science

Instantaneous speed and velocity and equations of motion

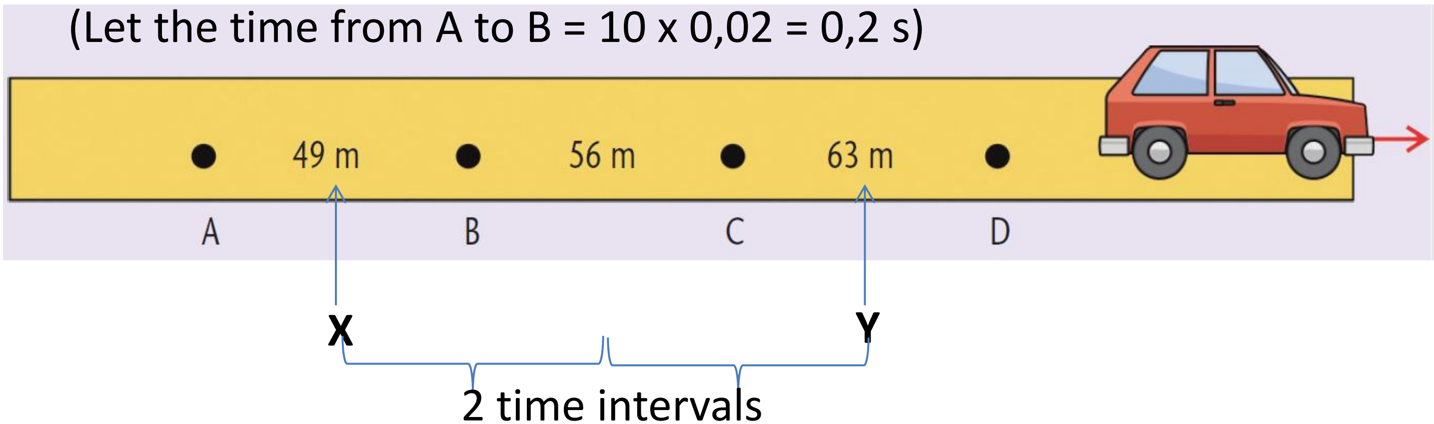
Instantaneous speed and velocity and equations of motion

<u>Definitions:</u>

Instantaneous velocity – the displacement divided by a VERY small time interval (vector).

Instantaneous speed – the <u>magnitude</u> of the instantaneous velocity (scalar).

Instantaneous velocity on a ticker tape in the middle of a time interval = the average velocity over the whole time interval.



Av. velocity over AB = $\frac{\Delta x}{\Delta t}$ = 49/0,2 = 245m.s^{-1} = v_i

Av. velocity over CD = $\frac{\Delta x}{\Delta t}$ = 63/0,2 = 315m.s^{-1} = v_f

Let X be a point in the middle of the **time** interval AB

Let Y be a point in the middle of the **time** interval CD

The time from X to Y form 2 intervals

Instantaneous velocity at X = 245 m.s^{-1}

Instantaneous velocity at Y = 315 m.s^{-1}

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{315\text{-}245}{2 \times 0{,}2} = 175\ m.s^{-2}$$

Application of instantaneous velocity

Remember: instantaneous velocity = average velocity (same formula)

See Textbook p. 325

Instantaneous velocity is the displacement travelled divided by a very small time interval. It is a vector quantity which means it has magnitude and direction. The same equation is used to find instantaneous velocity as was used to find average velocity:

$$\Delta v = \frac{\Delta x}{\Delta t}$$

Δv = velocity ($m.s^{-1}$)
Δx = displacement (m)
Δt = time (s)

Classroom activity 1

A car is travelling on the freeway in an area where the speed limit is 120 $km.h^{-1}$. The car approaches a traffic camera and the camera records that the car travels 0,36 m in 0,01 s.

1. What is the instantaneous speed of the car in $m.s^{-1}$?
2. Convert your answer in a) to $km.h^{-1}$.
3. Will the car get a speeding fine? Explain.

Describing motion i.t.o. words, diagrams and graphs

Constant velocity:

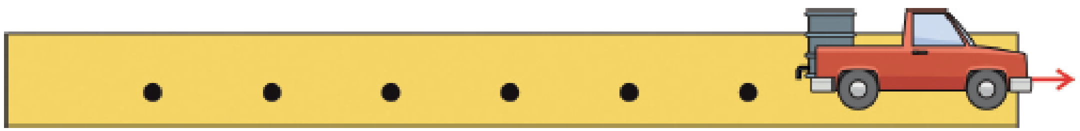


Fig 21.1 Truck travelling with a constant velocity

Because the truck is travelling at a constant velocity, its acceleration will be zero as it is not getting any faster or any slower.

Figure 21.2 shows the motion of the truck travelling at a constant velocity on graphs showing velocity, displacement and acceleration against time.

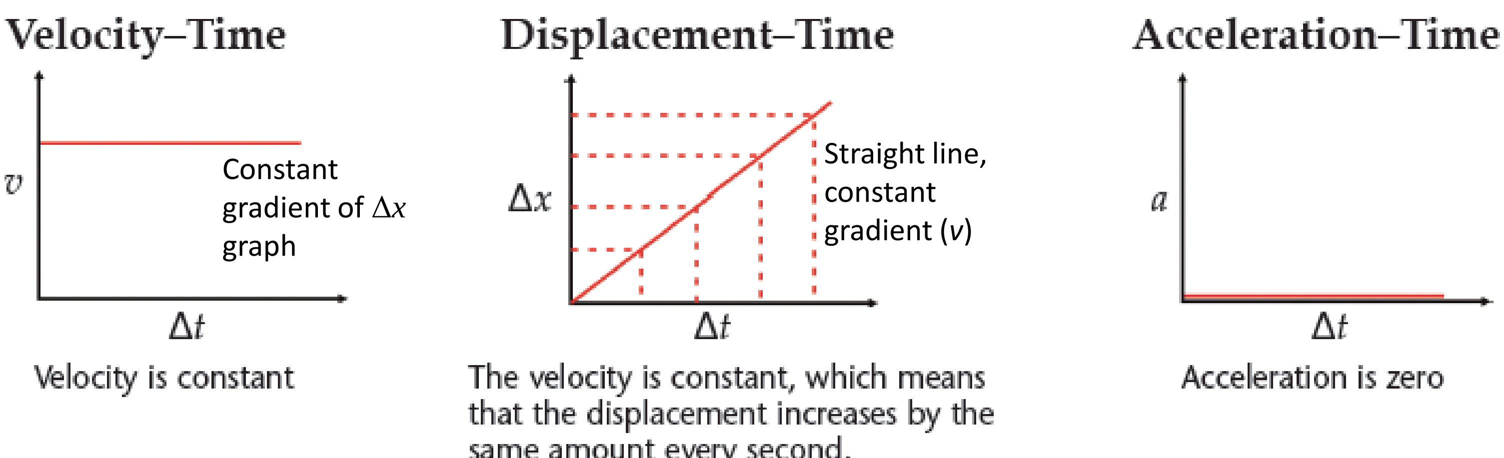


Fig 21.2 Graphs to describe characteristics of uniform motion

Describing motion i.t.o.
Words, diagrams and graphs

Accelerated motion:

An object that is accelerating uniformly

An object that is accelerating uniformly is speeding up by the same amount every second. This means that if the object starts from rest and accelerates at 2 m.s^{-2}, its velocity will increase by 2 m.s^{-1} every second.

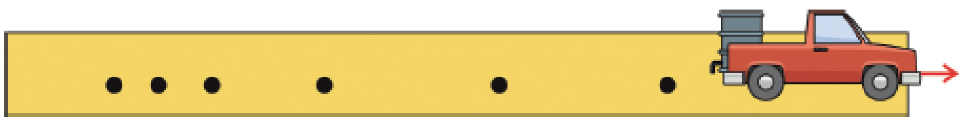
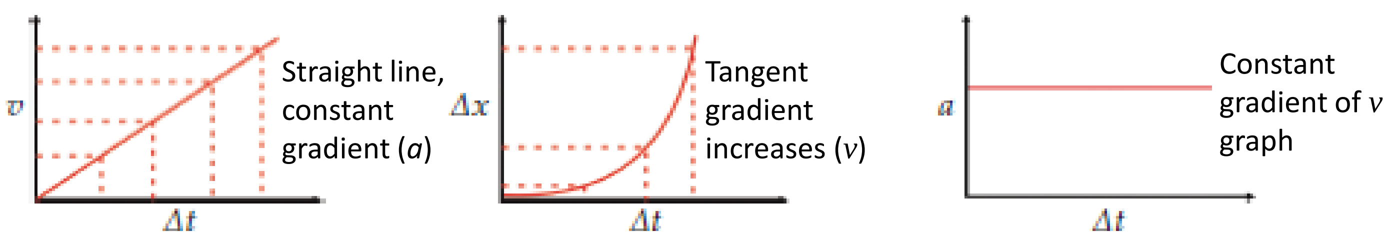


Fig 21.3 A uniformly accelerating truck

Figure 21.4 shows the motion of the truck accelerating uniformly from rest on graphs showing velocity, displacement and acceleration against time.

Velocity–Time | **Displacement–Time** | **Acceleration–Time**



The acceleration is uniform, so the velocity increases by the same amount every second.

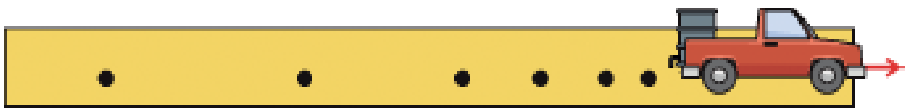
The velocity is increasing. Therefore the displacement covered each second increases.

The acceleration is uniform or constant.

Fig 21.4 Graphs to describe characteristics of uniform acceleration

Describing motion i.t.o. Words, diagrams and graphs

Decelerated motion:

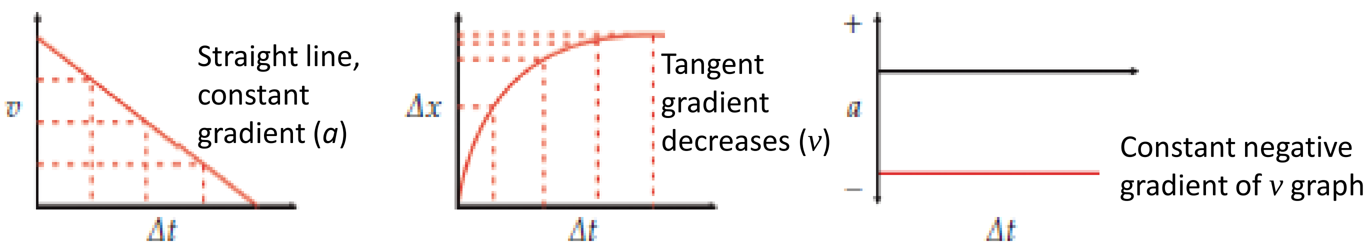


21.5 A truck slowing down

Figure 21.5 again shows the truck, but this time it is decelerating uniformly at 2 m.s^{-2}. This means that it will slow down up by 2 m.s^{-1} every second and that the distance travelled each second will decrease.

Figure 21.6 shows the motion of the truck decelerating uniformly on graphs showing velocity, displacement and acceleration against time.

Velocity–Time | **Displacement–Time** | **Acceleration–Time**



The deceleration is uniform, so the velocity decreases by the same amount every second.

The velocity is decreasing. Therefore the displacement covered each second decreases.

The deceleration is uniform, or constant. The acceleration is opposite to the direction of motion, therefore has a negative value.

Fig 21.6 Graphs to describe characteristics of uniform deceleration

Graph examples

Remember:

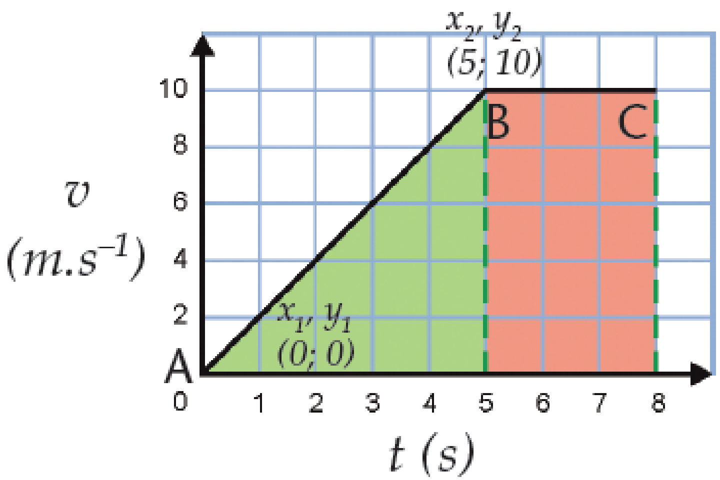
1. On a velocity–time graph:
Displacement can be found by calculating the area under the graph.
Acceleration can be found by calculating the gradient of the graph.

2. On a displacement–time graph:
Instantaneous velocity can be found by calculating the gradient of the graph.

Graph examples

Worked examples

1 The velocity–time graph of a car moving west away from a traffic light when it turns green is shown in Figure 21.7.



Describe motion:

- Start from rest (if v = 0 m.s^{-1})
- Constant acceleration for 5 seconds (gradient)
- Reaches constant velocity of 10 m.s^{-1} at 5 seconds.
- Moves at constant velocity for 3 seconds

REMEMBER:

Displacement = Δx = Area under graph = *l* x b = m.s^{-1} x s^{1} = m

acceleration = *a* = gradient of graph = rise over run = $^{\Delta v}/_{\Delta t}$ = m.s^{-1} / s = m.s^{-2}

Follow the solution on p. 328

Graph examples (continue)

2 The velocity–time graph of a car moving west away from a traffic light when it turns green is shown in Figure 21.8.

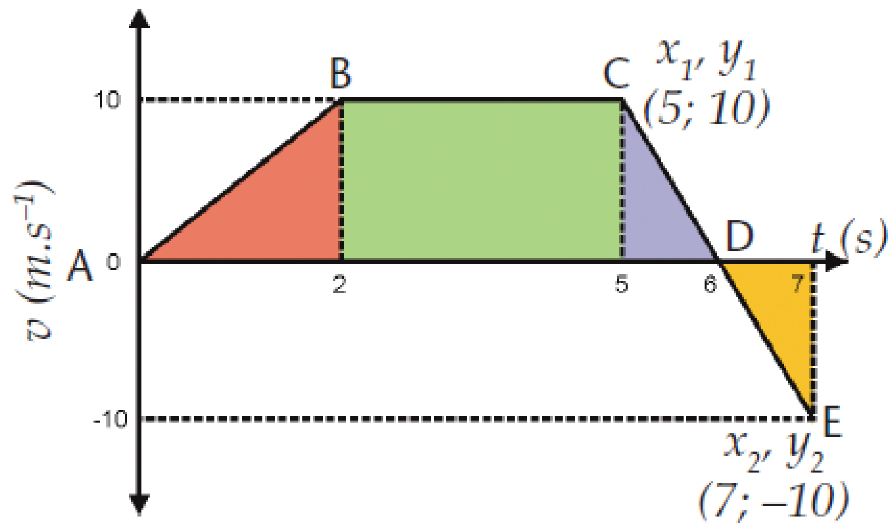


Fig 21.8 Velocity–time graph of a car

The positive and negative axes on this graph indicate direction. The car is initially travelling west and this motion is shown on the positive axis, A to D. From D to E the car travels east, so this is shown on the negative axis.

Describe motion:

- 0-2s: Starts from rest and constant acceleration for 2 seconds (gradient) westerly
- 2-5s: Reaches constant velocity of 10 m.s^{-1} at 2 seconds and maintains velocity west.
- 5-6s: Negative acceleration and stop west
- 6-7s: Constant acceleration to reach velocity 10m.s^{-1} east (opposite direction: velocity graph goes negative)

Follow solution on p. 329 do HW p. 335 no. 1,2

Graph examples

3 Figure 21.9 shows the displacement–time graph for a train travelling south along a train track.

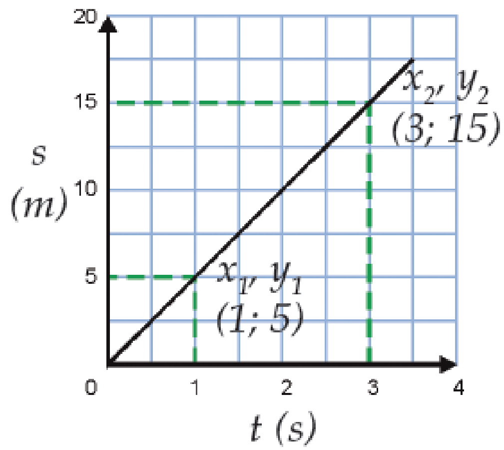


Fig 21.9 Displacement–time graph of a train travelling south

a) Describe the motion of the train.

The train is travelling south at a constant velocity.

b) Calculate the velocity of the train at 3 s.

$$\text{Velocity} = \text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15-5}{3-1} = \frac{10}{2} = 5\ m.s^{-1}\text{south}$$

The velocity of the train is constant, so any two points on the line can be used to calculate the gradient of the line.

Graph examples

4 Figure 21.10 shows the displacement–time graph of car travelling along a road in a westerly direction.

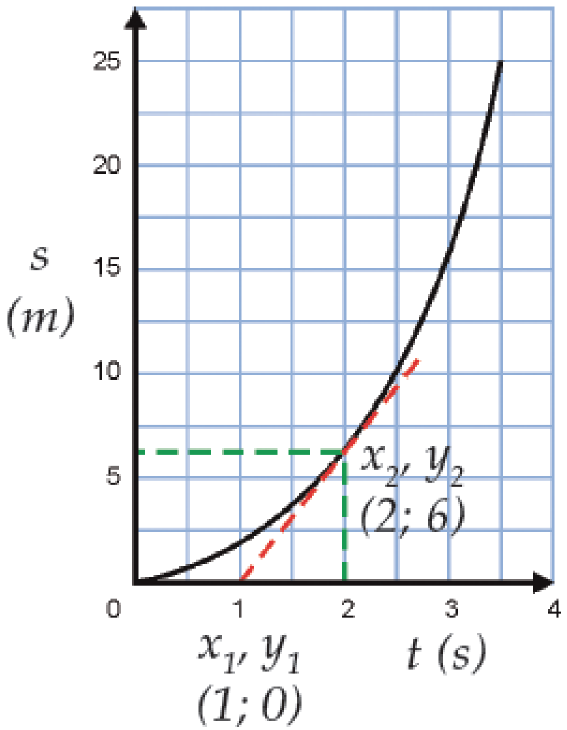


Fig 21.10 Displacement–time graph of a car in a westerly direction

a) Describe the motion of the car.

The car is accelerating uniformly to the west.

b) Calculate the velocity of the car at 2 s.

$$Velocity = gradient = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{2-1} = \frac{6}{1} = 6\ m.s^{-1}\ west$$

The velocity of the car is increasing as it is accelerating. In order to calculate the velocity at a point on this curved graph, we need to draw a tangent to the curve at that point and then calculate the gradient of that tangent.

HW: Do p. 336 no. 1,2,3 (no. 4 will be class work)

Equations of motion

$$v_f = v_i + a\Delta t$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$\Delta x = \left(\frac{v_1 + v_f}{2}\right)\Delta t$$

v_i = initial velocity ($m.s^{-1}$)
v_f = final velocity ($m.s^{-1}$)
Δx = distance / displacement (m)
Δt = time (s)
a = acceleration ($m.s^{-2}$)

See examples and exercises in text book

Equations of motion cnt p. 338

Worked examples

1 A car travelling at 4 $m.s^{-1}$ east is accelerated uniformly at 3 $m.s^{-2}$.

a) What distance will the car travel in 5 s?

$v_i = 4\ m.s^{-1}$
$a = 3\ m.s^{-2}$
$\Delta t = 5\ s$
$\Delta x = ?$

$$\begin{aligned}\Delta x &= v_i\Delta t + \tfrac{1}{2}a\Delta t^2 \\ &= (4)(5) + \tfrac{1}{2}(3)(5^2) \\ &= 57{,}5\ m\end{aligned}$$

b) Calculate the velocity after 5 s.

$v_i = 4\ m.s^{-1}$
$a = 3\ m.s^{-2}$
$\Delta t = 5\ s$
$\Delta x = 57{,}5\ m$
$v_f = ?$

$$\begin{aligned}v_f &= v_i + a\Delta t \\ &= 4 + (3)(5) \\ &= 19\ m.s^{-1}\ east\end{aligned}$$

2 An aircraft, flying at an unknown initial velocity in an easterly direction, accelerates uniformly at 5 $m.s^{-2}$. It reaches a velocity of 200 $m.s^{-1}$ east after accelerating over a distance of 300 m. Calculate the aircraft's initial velocity.

$a = 5\ m.s^{-2}$
$v_f = 200\ m.s^{-1}$
$\Delta x = 300\ m$
$v_i = ?$

$$\begin{aligned}v_f^2 &= v_i^2 + 2a\Delta x \\ 200^2 &= v_i^2 + 2(5)(300) \\ v_i^2 &= 37\ 000 \\ v_i &= 192{,}35\ m.s^{-1}\ east\end{aligned}$$