WORK, ENERGY and POWER

- MECHANICS -

(Topic 4)

Recap…

- **Gravitational potential energy** (E_p – measured in Joules (J))
  - The energy an object has due to its position (in the gravitational field) relative to a reference point (usual ground level)
  - $\boxed{E_p = mgh}$ **where:** m = mass (kg)
    g = gravitational acceleration ($9.8 m.s^{-2}$)
    h = vertical height above the reference point (m)

- **Kinetic energy** (E_k – measured in Joules (J))
  - The energy an object has due to its motion
  - $\boxed{E_k = \frac{1}{2} mv^2}$ **where:** m = mass (kg)
    v = speed ($m.s^{-1}$)

- **Mechanical energy** (E_M – measured in Joules (J))
  - The sum of the gravitational energy and kinetic energy of an object
  - $\boxed{E_M = E_p + E_k}$

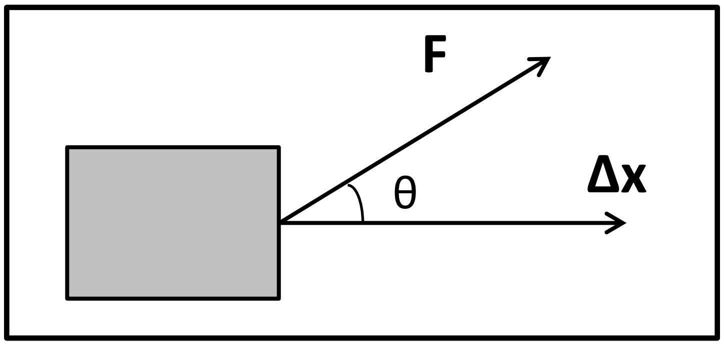
- **Conservation of Mechanical energy**
  - The total mechanical energy of an object remains constant in an isolated system
  - **Therefore…** if there are no external forces (eg. Friction) acting on an object, the mechanical energy of the object will be the same everywhere.

<u>WORK</u> (W)

If a constant **<u>force</u> (F)** acts on an object while it undergoes a **<u>displacement</u> (Δx)**, then the **<u>work done</u> (W)** is:

$$\boxed{W = F.\Delta x.\cos\theta}$$

<u>Where</u>: F = force (N)
Δx = displacement (m)
θ = angle between F and Δx



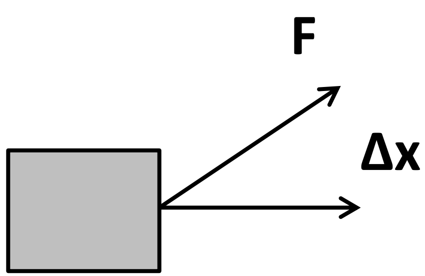
<u>Note</u>:
- Work is measured in JOULES (J)
- Work is a SCALAR (has magnitude only), but work can be POSITIVE or NEGATIVE

- Whenever work is done on an object by a force, ENERGY IS TRANSFERRED from one object to another. The amount of work done is equal to the energy transfer taking place.

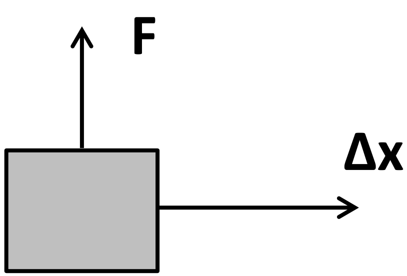
<u>Example 1</u>: Calculate the work done by a 100N force which acts at 20^0 to the horizontal. The force displaces the object 3m along a horizontal frictionless surface.

<u>Question</u>: How much energy is transferred to the object by the force?

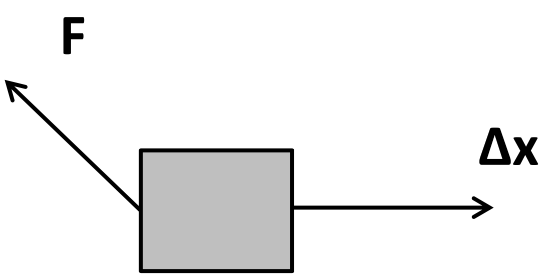
- **Positive vs. negative work**
  - Whether energy is transferred to an object (**or** removed from an object) depends on the **ANGLE (θ) BETWEEN THE FORCE AND DIRECTION OF THE DISPLACEMENT!**
  - If the force and direction of the displacement is…
    - in the SAME DIRECTION ($0 \leq \theta < 90$) → **POSITIVE** work will be done
    - at right angles to one another ($\theta = 90$) → **ZERO** work is done
    - in the OPPOSITE DIRECTION ($90 < \theta \leq 180$) → **NEGATIVE** work will be done



POSITIVE WORK



ZERO WORK



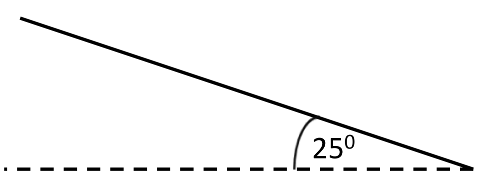
NEGATIVE WORK

Example 2: An athlete attaches a small parachute directly behind her to increase the force of air resistance acting on her during training. The athlete experiences a force of 80N while she runs. Calculate the work done on the athlete by the 80N force is she runs a distance on 30m.

- **Net work done** **(W_{NET})**
  - The Net work on an object is the overall **work done by a number of different forces** being exerted on the object.
  - There are 2 methods for determining the net work done (W_{NET})

1) Calculating the work done by EACH force
   - Draw a force diagram of all forces acting ON the object
   - Determine the angle θ between each force and the direction of the displacement
   - Use: $W = F\Delta x\cos\theta$ to find the work done by EACH force
   - Add the work done by each force to obtain the NET work done

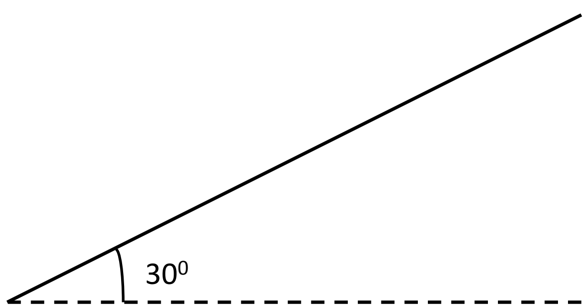
Example 3: A boy drives his 3kg remote-controlled car up a 4m long plane which is inclined at 25^0 to the horizontal. The cars motor exerts an average forward force of 40N. The car experiences a frictional force of 15N as it moves up the incline. Calculate the net work done on the car.



2) <u>Calculating the work done by the NET force</u>

- Draw a force diagram showing all the forces acting ALONG the plane of motion
- Ignore forces acting perpendicular to the plane (because they do zero work on the object)
- Calculate the NET FORCE (F_{NET}) acting on the object PARALLEL to the plane
- Calculate the work done by the net force (ie. The NET work) using: $W_{NET} = F_{NET}\Delta x\cos\theta$

Example 4: A Truck of mass 2000kg free wheels down a 30m long inclined plane. The plane is inclined at 30^0 to the horizontal. The truck experiences a constant frictional force of 5500N as it free wheels down the inclined plane.

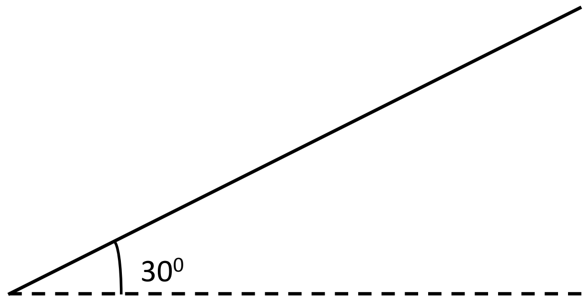


Note: Whenever POSITIVE net work is done on an object, the energy of the object will INCREASE
Whenever NEGATIVE net work is done on an object, its energy will DECREASE
The energy gained/lost is generally KINETIC ENERGY (work-energy theorem)

- **WORK-ENERGY THEOREM**
  - "The **NET work** done on an object is equal to the **CHANGE IN KINETIC ENERGY** of the object"
  - ie: $\boxed{W_{NET} = \Delta E_K}$ (where: $\Delta E_k = E_{kf} - E_{ki}$)

Example 5: A formula 1 racing car of mass 640kg is travelling at 30m.s^{-1}. It then accelerates in a straight line down the main straight. The engine exerts an average force of 12000N and the racing car experiences an average frictional force of 3000N. USING THE WORK-ENERGY THEOREM, calculate the speed of the racing car after travelling 30m

Example 6: A dynamics trolley of mass 2kg is held at the top of the plane, inclined at 30^0 to the horizontal. The trolley is released (from rest) and then rolls down the plane while experiencing a constant frictional force of 6N. USING THE WORK-ENERGY THEOREM, calculate the speed of the trolley after it has rolled 1.5m down the plane.

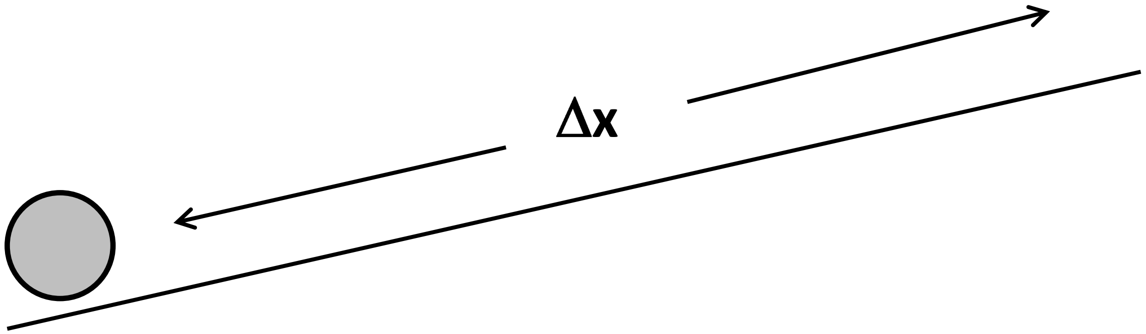


- **CONSERVATION OF ENERGY WHEN NON-CONSERVATIVE FORCES ARE PRESENT**
  - CONSERVATIVE FORCES
    - A force is conservative if the NET WORK done by the force is ZERO, while moving an object around a CLOSED PATH (starting and ending at the same point).
    - Example: A 1kg ball thrown vertically up, reaching a maximum height of 2m
      - The only force is the gravitational force (F_g) acting during the motion
      - The path travelled is closed
      - Work done (UP): $W_{UP} = F_g\Delta x\cos\theta = mg\Delta x\cos\theta = (1)(9.8)(2)\cos180^O = -19.6J$
      - Work done (DOWN): $W_{DOWN} = F_g\Delta x\cos\theta = mg\Delta x\cos\theta = (1)(9.8)(2)\cos0^O = +19.6J$
      - Therefore, $W_{NET} = -19.6 + (+19.6) = 0J$
      - Therefore, The gravitational force is conservative

NOTE: WHEN CONSERVATIVE FORCES are present, TOTAL MECHANICAL ENERGY is CONSERVED

- If the ball was thrown higher, the NET WORK done by the gravitational force will still be zero, because the **NET WORK done by a conservative force is path INDEPENDANT**

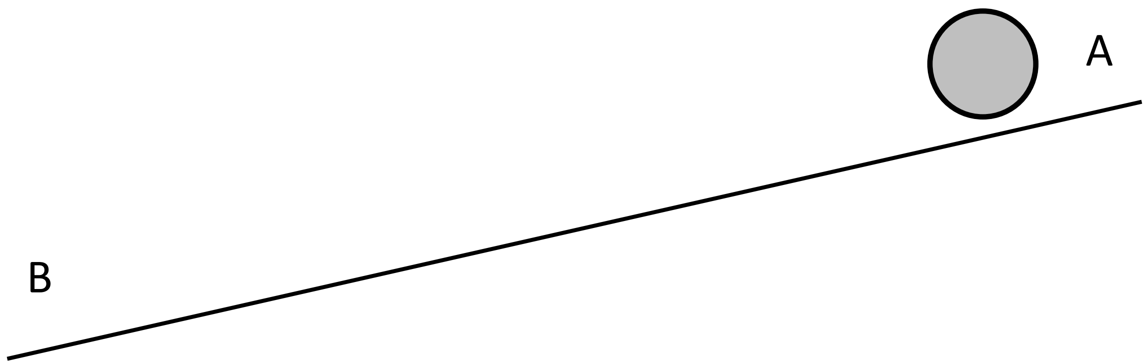
- **NON** -CONSERVATIVE FORCES
  - Consider a ball rolled 5m up a rough slope, that then rolls back to its starting point



  - All throughout the motion of the ball, a FRICTIONAL force (say 30N) acts on the ball
  - The friction is always in the opposite direction to the motion
  - Work done by friction (UP): $W_{UP} = f\Delta x\cos\theta = (30)(5)\cos180^{O} = -150J$
  - Work done by friction (DOWN): $W_{DOWN} = f\Delta x\cos\theta = (30)(5)\cos180^{O} = -150J$
  - Therefore, $W_{NET} = -150 + (-150) = -300J$
  - Therefore, $W_{NET} \neq 0$ and thus FRICTION is a **NON**-CONSERVATIVE FORCE
  - This will result in the total mechanical energy NOT being conserved
  - Also, the NET work done by friction is PATH DEPENDANT, because if the ball was made to move further (ie. more than 5m), there would be a greater net negative work done

NOTE: Applied forces are also NON-conservative (They increase total E_M, due to positive work)

- <u>WORK DONE BY A **NON** -CONSERVATIVE FORCE</u>
  - Suppose a trolley rolls down a slope from point A to B



  - If friction acts on the trolley, negative work will be done by the non-conservative force
  - W_{NC} results from the force of friction
  - Some of the mechanical energy will be "lost" due to this non-conservative force
  - BUT... According to the law of conservation of energy, energy cannot be destroyed
  - So the work done by the non-conservative force must be included...

$$E_{k(i)} + E_{p(i)} = E_{k(f)} + E_{p(f)} - W_{NC}$$

$$W_{NC} = E_{k(f)} + E_{p(f)} - E_{k(i)} - E_{p(i)} = E_{k(f)} - E_{k(i)} + E_{p(f)} - E_{p(i)}$$

$$\boxed{\mathbf{W_{NC} = \Delta E_k + \Delta E_P}}$$

Example 7: A 70kg skateboarder moves down a slope while experiencing a frictional force of 190N. The slope forms an angle of 30^{o} with the horizontal. At some point “A” on the slope, the skateboarder is moving at $6 m.s^{-1}$. 10m after point A, he reaches the bottom of the slope (point B).

Calculate: a) The gravitational potential energy of the skateboarder at point A

b) The speed of the skateboarder at point B

- **POWER**
  - “The **RATE** at which **WORK** is done” (Note: Work is a scalar)
  - ie: $$\boxed{P = \frac{W}{t}}$$ where: P = Power (watts, W)

    W = Work (J)

    t = time (s)

Example 8: A young boy, mass 60kg, runs to the top of a 3m high flight of stairs. If he takes 10 seconds to reach the top of the stairs while starting from rest and travelling at a speed of 3 $m.s^{-1}$ the top, calculate the power output of the boy.

Example 9: An Olympic sprinter of mass 75kg is capable of reaching a speed of $10m.s^{-1}$ from rest in $3,5m.s^{-1}$. Calculate the average power output of the sprinters leg muscles.

Example 10: The engine of a 300kg dragster exerts an average forward force of 53000N over the first 20m of its race. If the dragster takes 0.8s to cover the first 20m, calculate the power developed by the engine.

- **AVERAGE POWER**
  - Consider a car moving at a CONSTANT SPEED along a straight road. The car's engine exerts a forward force (F) on the car. The car also experiences a frictional force (*f*) in the opposite direction. For CONSTANT speed, the forces need to balance (ie. F = *f*), because: $F_{NET} = ma$
  - The work done by the forward force is given by: W = F.Δx.cosθ = F.Δx
  - Therefore: $P_{\text{av}} = \frac{W}{t} = \frac{F\Delta x}{t} = F\left(\frac{\Delta x}{t}\right) = Fv_{\text{av}}$

$$\boxed{\mathbf{P_{av} = F.\mathit{v}_{av}}}$$

  - Thus, The average power required to keep an object moving at a CONSTANT speed is found by multiplying the APPLIED force (F) by the average speed of the object (v_{av})

Example 11: A motor cyclist is travelling at a constant speed of $80 m.s^{-1}$ on a super bike. The engine produces a forward force of 1800N. Calculate the average power produced by the engine.

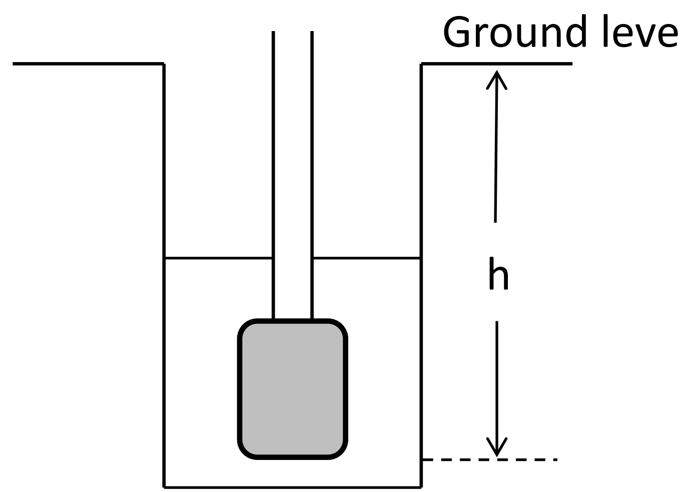
Example 12: A locomotive engine of mass 20000kg, which has an output power of 175kW, pulls carriages of total mass 40000kg at a maximum constant speed of 55m.s^{-1} along a straight level track. Calculate the magnitude of the forward force of the train's engine.

Question: What is the magnitude of the total resistive force opposing the motion

Question: If the engine is switched off and the brakes are now applied, calculate the power of the braking system if the train is brought to rest in 36,4s over a distance of 1000m.

Example 13: A cyclist rides up a rough inclined plane at a constant speed of $3 m.s^{-1}$. The combined mass of the cyclist and bike is 90kg. If the cyclist experiences a frictional force of 250N, Calculate the average power of the cyclist.

- **POWER OF AN ELECTRIC MOTOR**
  - Consider An electric pump used to pump water out of a bore hole. The pump will pull water through a vertical height, h at a certain rate (usually given in litres per minute, $L.min^{-1}$)



Example 14: Calculate the minimum power required of an electric motor pump to pump water at a rate of 120 litres per minute from a depth of 12m

Note: - The upward force exerted by the pump is a NON-conservative force
- The motor pumps water at a CONSTANT RATE (ie. At a constant speed), thus $\Delta E_K = 0$
- 1L of water has a mass of 1kg